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Tangents Method to Profile Rack-Tools to Generate by Wrapping

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ABSTRACT

Modelling surfaces generated by wrapping, by using the rolling method, supposes to know a rack-tool profile numerical model together to a specific algorithm to study the enwrapped of a profiles family discretely expressed. On this way, a specific algorithm – "Tangents Method" – was created, in order to allow enwrapped surfaces (profiles) study, when profile to be generated is discretely represented. The quality of this method was proved by comparing the results from its application to the solutions given by already known methods. Examples when generating by using a rack-tool an elementary profile (a straight line segment) are presented. The results obtained show the quality of suggested algorithm to study enwrapped profiles associated to a couple of rolling centrods.
***Keywords**: enwrapped surfaces, rolling centrods, tangents.*

1. Generalities

"Tangents Method" verification as a way to realize profiles discrete representation is suggested to be done by comparing the results referring to tools profiles, obtained by both fundamental and complementary known theorems, when generating the same whirl of surfaces.

Thus, it is suggested to use a fundamental theorem ("Willis's Theorem")[1],[3] and three complementary methods ("Minimum Distance Method"[4], "Plain Generating Trajectories Method"[7] and "Substituting Circles Family Method"[4]) to profile the rack-tool that generates the same profile, discretely expressed[6].

2. Rectilinear Profile – "Tangents Method"

A certain profile, from the whirl of profile to be generated, associated to a circular centrod having the radius R_{rp}, can be discretely expressed through the matrix (1), referred to XY system, associated to the whirl of profiles to be generated.

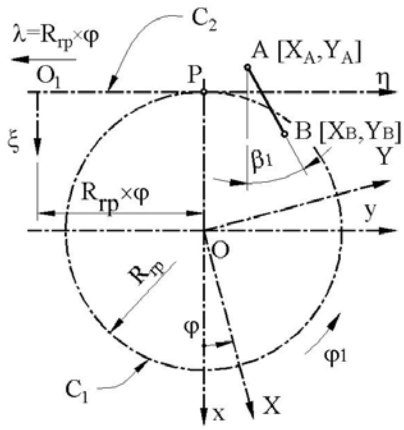

***Figure 1**. The whirl of profiles to be generated*

The number of co-ordinates to define the matrix (1) can be correlated to the precision of representing the Σ profile [5],[8],

$$\Sigma = \begin{Vmatrix} X_A & Y_A \\ X_2 & Y_2 \\ \vdots & \vdots \\ X_i & Y_i \\ \vdots & \vdots \\ X_B & Y_B \end{Vmatrix}. \quad (1)$$

The model used to define Σ matrix current point co-ordinates is:

$$\begin{cases} X = X_A + v \cdot \cos \beta_i; \\ Y = Y_A + v \cdot \sin \beta_i, \end{cases} \quad (2)$$

where v is an incremental variable.

The Σ family of profiles, described into the $\xi\eta$ reference system, associated to the worked piece, has the form:

$$\begin{vmatrix} \xi = X_i \cdot \cos \varphi_1 - Y_i \cdot \sin \varphi_1 + R_{rp}; \\ \eta = X_i \cdot \sin \varphi_1 + Y_i \cdot \sin \varphi_1 + R_{rp} \cdot \varphi_1. \end{vmatrix} \quad (3)$$

The enveloping condition, determined to express generated profile in the form (1), by using tangents method, is

$$\left| \sin \beta_i \left[Y_i - R_{rp} \sin \varphi_1 \right] - \cos \beta_i \left[-X_i - R_{rp} \cos \varphi \right] \right| \le \varepsilon, \quad (4)$$

where ε is small enough.

The ensemble formed by the family of profiles discrete representation (3) and the specific enveloping condition (4) represents, under discrete form, the profile of rack-tool enwrapped to Σ profile.

Based on the algorithm briefly upper suggested a flowchart, see Figure.2 was imagined together to an application written into *java* language, to allow us to find, by starting from a numerical expression of profile to be generated, the rack-tool profile.

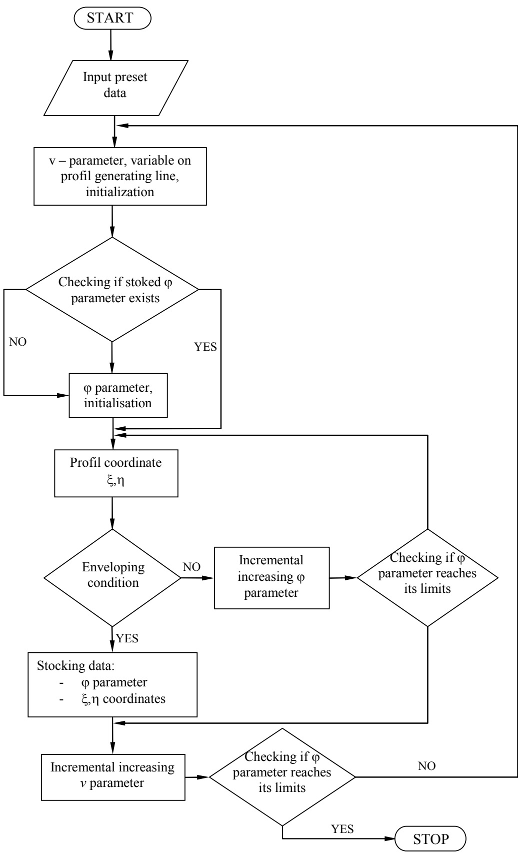

***Figure 2**. Flowchart*

In figure 3 and table1 (extracted from 300 points co-ordinates) rack-tool shape and the co-ordinates of its profile, to generate a profile having the characteristics $A[X_A = -60, Y_A = 0]$; $B[X_B = -50,\ Y_A = 10]$; $R_{rp} = 60$ mm, are shown.

***Table1.** Rack-tool Profile, Tangents Method*

ξ[mm]	η[mm]
0.00000	0.00000
0.03345	0.03349
0.06690	0.06705
0.10036	0.10069
0.13381	0.13440
……..	….....
9.19720	12.62780
9.22421	12.67817
9.25079	12.72785

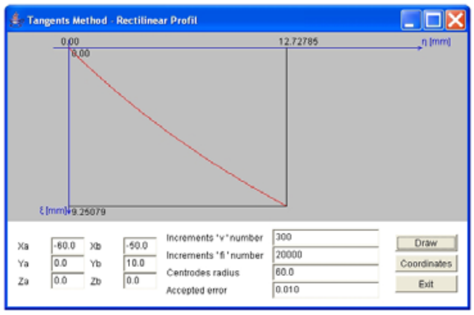

***Figure 3.** Rack-tool shape;Jjava applet*

3. Rectilineal Profile – "Minimum Distance" Method

If using the same representation of Σ profile,

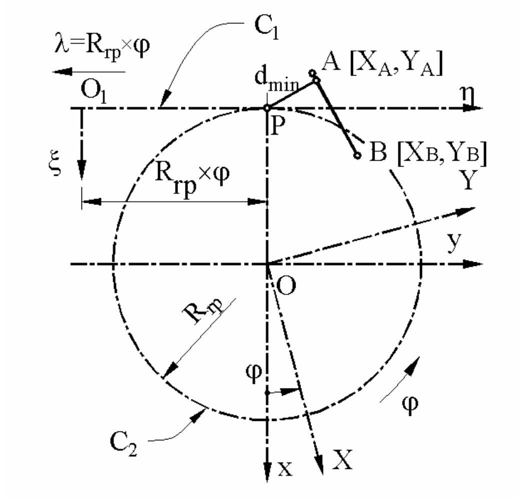

***Figure 4**. Minimum distance method*

from the whirl associated to C_2 centrod, "Minimum Distance Method" leads to a profiles family, referred to rack-tool reference system (ξη, associated to C_1 centrod), given as:

$$\begin{cases} \xi_i = X_i \cdot \cos\varphi_1 - Y_i \cdot \sin\varphi_1 + R_{rp}; \\ \eta_i = X_i \cdot \sin\varphi_1 + Y_i \cdot \sin\varphi_1 + R_{rp} \cdot \varphi_1, \end{cases} \quad (5)$$

and also to "Minimum Distance Method" specific enveloping condition

$$d = \left\{ \sqrt{\left(\xi_i - \xi_P\right)^2 + \left(\eta_i - \eta_P\right)^2} \right\}_{\text{minimum}} \quad (6)$$

where ξ_P, η_P are the gearing pole co-ordinates:

$$\xi_P = 0;\ \eta_P = R_{rp} \cdot \varphi_1, \quad (7)$$

so (6) form becomes

$$d = \left\{ \sqrt{\xi_i^2 + \left(\eta_i - R_{rp} \cdot \varphi_1\right)^2} \right\}_{\text{minimum}} . \quad (8)$$

The ensemble given through (4) and (8) representations constitutes the rack-tool profile (discrete form). By using the new algorithm, specific to "Minimum Distance Method", to discretely express the minimum condition, a flowchart, similar to the one from "Tangents Method", see figure.2, was imagined; the difference results from the specific forms of profile to be generated equations and of enveloping condition. Starting from here, another *java* application was conceived, to allow rack-tool profile numerical and graphical expression.

In figure 5 and table 2 (extracted from 2000 points), rack-tool profile co-ordinates are

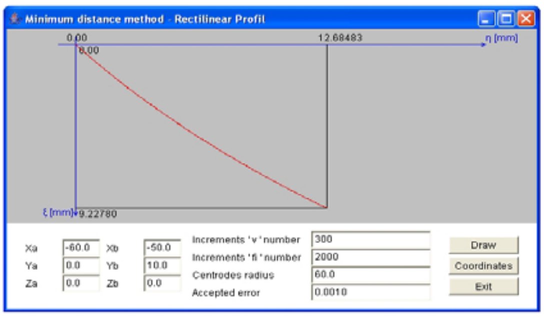

Figure 5. *Minimum distance method;Java applet*

shown, if generating the same upper profile.

Table2. *Rack-tool Profile, Minimum Distance Method*

ξ[mm]	η[mm]
0.00000	0.00000
0.43484	0.44117
0.90205	0.92947
1.33595	1.39643
....	....
8.35056	11.10189
8.81632	11.92888
9.22780	12.68483
9.25079	12.72785

4. Rectilinear Profile – "Plain Generating Trajectories" Method

To do AB segment analytical representation, see figure6, the model suggested to define Σ

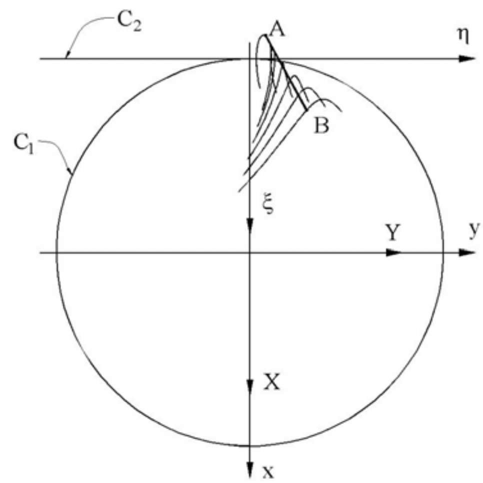

Figure 6*. Plain generating trajectories method*

matrix current point co-ordinates is similar to (2), when β_i means an imposed constant and v – an incremental variable. The profiles family results as presented in relation (9), similar to (3) form; variables v and φ_1 are treated, in this case, as continue, independent variables:

$$\begin{cases} \xi = \left[X_A + v\cos\beta_i\right]\cos\varphi_1 - \left[Y_A + v\sin\beta_i\right]\sin\varphi_1 + R_{rp}; \\ \eta = \left[X_A + v\cos\beta_i\right]\sin\varphi_1 + \left[Y_A + v\sin\beta_i\right]_i \sin\varphi_1 + R_{rp}\varphi_1. \end{cases} \quad (9)$$

Partial derivatives can be found, starting from (9):

$$\begin{cases} \dfrac{d\xi}{dv} = \cos(\varphi_1 + \beta_i); \\ \dfrac{d\eta}{dv} = \sin(\varphi_1 + \beta_i); \\ \dfrac{d\xi}{d\varphi_1} = -\left[X_A + v\cos\beta_i\right]\sin\varphi_1 - \left[Y_A + v\sin\beta_i\right]\cos\varphi_1; \\ \dfrac{d\eta}{d\varphi_1} = \left[X_A + v\cos\beta_i\right]\cos\varphi_1 - \left[Y_A + v\sin\beta_i\right]\sin\varphi_1 + R_{rp}, \end{cases} \quad (10)$$

so the specific enveloping condition can now be written:

$$\left| \begin{matrix} \dfrac{d\xi}{dv} & \dfrac{d\eta}{dv} \\ \dfrac{d\xi}{d\varphi_1} & \dfrac{d\eta}{d\varphi_1} \end{matrix} \right| \le \varepsilon, \quad (11)$$

if ε is arbitrary and small enough, correlated to desired precision of rack-tool finding.

Based on this algorithm, a dedicated flowchart was imagined and a *java* program to

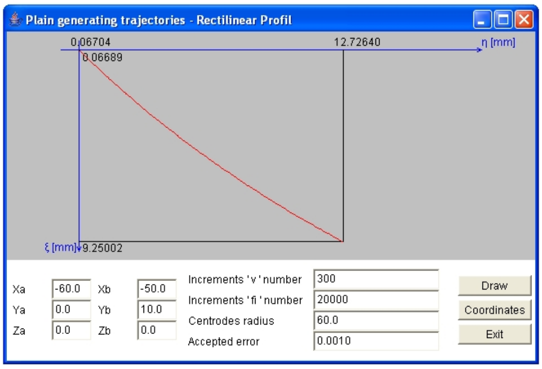

***Figure 7**. Plain generating trajectories method; Java applet*

find, numerical and graphical, rack-tool profile form was written. In figure 7 and table 3, rack-tool profile shape and its points co-ordinates are presented, when generating a profile having same characteristics as above.

***Table 3.** Rack-tool Profile, Plain Generating Trajectories Method*

ξ[mm]	η[mm]
0.00000	0.00000
0.03345	0.03349
0.06690	0.06705
0.10036	0.10069
0.13381	0.13440
………..	………..
9.19720	12.62780
9.22421	12.67817
9.25079	12.72785

5. Rectilinear Profile – "Substituting Circles Family" Method

As it follows from "Substituting Circles Family" method, the family of circles

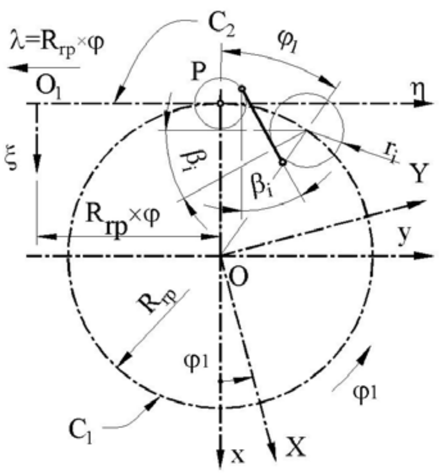

***Figure 8**. Substituting circles family method*

associated to AB segment is defined through:

$$\begin{cases} X = -R_{rp} \cdot \cos\varphi_1 + r_i \cdot \sin\beta_i; \\ Y = R_{rp} \cdot \sin\varphi_1 - r_i \cdot \cos\beta_i, \end{cases} \tag{12}$$

where r_i, β_i can be defined from condition that family circles must be tangent to AB segment, see (2) equations.

This condition allows us to find the identification equations:

$$\begin{aligned} X_A + v\cdot\cos\beta_i + R_{rp}\cdot\cos\varphi_1 &= r_i\cdot\sin\beta_i; \\ Y_A + v\cdot\sin\beta_i - R_{rp}\cdot\sin\varphi_1 &= -r_i\cdot\cos\beta_i, \end{aligned} \tag{13}$$

which represents the conditions of common points between circles and segment ;

$$\begin{cases} \dfrac{dX}{dv} = r_i\cdot\cos\beta_i; \\ \dfrac{dY}{dv} = r_i\cdot\sin\beta_i, \end{cases} \tag{14}$$

which represents common tangent, in contact point, condition.

Thus, they can be defined:

$$r_i = \left\{\left[X_A + v\cdot\cos\beta_i + R_{rp}\cdot\cos\varphi_1\right]^2 + \right.$$
$$\left. + \left[Y_A + v\cdot\sin\beta_i - R_{rp}\cdot\sin\varphi_1\right]^2\right\}^{1/2} \tag{15}$$

$$tg\beta_i = \frac{|Y_{i+1} - Y_i|}{|X_{i+1} - X_i|}; \tag{16}$$

and the condition

$$\left|\frac{\dfrac{dY}{dv}}{\dfrac{dX}{dv}} - \frac{X_{(v)} + R_{rp}\cdot\cos\varphi_1}{Y_{(v)} - R_{rp}\cdot\sin\varphi_1}\right| \le \varepsilon. \tag{17}$$

The last relation represents the enveloping condition specific to "Substituting Circles Family" method. The family of substituting circles, transposed into rolling motion to C_2 centrod, has the equations:

$$\begin{aligned} \xi_i &= r_i \sin(\varphi_1 + \beta_i); \\ \eta_i &= -r_i \cos(\varphi_1 + \beta_i) + R_{rp}\varphi_1. \end{aligned} \tag{18}$$

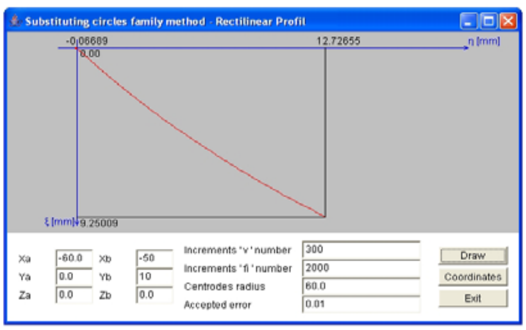

***Figure 9**. Substituting circles family method;Java applet*

The ensemble of (15), (16), (17) and (18) equations represents rack-tool profile, enwrapped to profile to be generated. "Substituting Circles Family" method specific scheme is shown in figure 9 and table 4.

Table 4. *Rack-tool Profile, Substituting Circles Family Method*

ξ[mm]	η[mm]
0.00000	0.00000
0.03344	-0.03344
0.06689	-0.06689
0.07478	-0.00746
0.09714	0.06773
………..	……….
9.16944	12.57614
9.19635	12.62621
9.22323	12.67635
9.25079	12.72785

6. Rectilinear Profile – Willis Method

This method represents a theoretical one, figure10, being the main reference term to estimate "Tangents Method" quality.

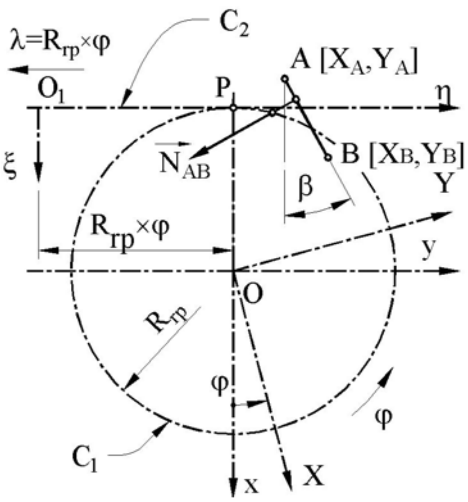

Figure 10. *Willis method*

To the family of profiles (3), the specific enveloping condition

$$\left|\left\{R_{rp}\cdot\cos\varphi_1-\left[X_A+v\cdot\cos\beta_i\right]\right\}\cdot\cos\beta_i+\left\{R_{rp}\cdot\sin\varphi_1-\left[Y_A+v\cdot\sin\beta_i\right]\right\}\cdot\sin\beta_i\right|\le\varepsilon \quad (19)$$

can be associated, where ε is arbitrary chosen and small enough, to ensure the desired precision to tool profile. The ensemble of equations (3) and (19) represents, in this case, the rack-tool profile.

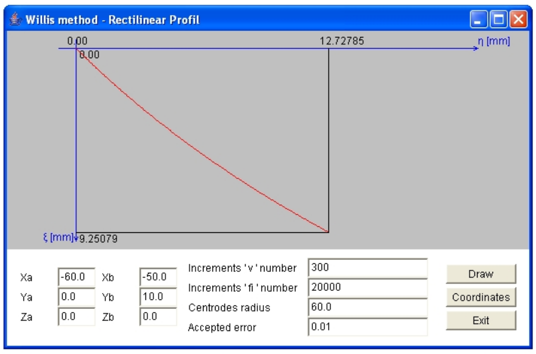

Figure 11*. Willis method; Java applet*

In Fig.11, the specific dedicated Java application is shown, while rack-tool profile points co-ordinates, found by using the mentioned application, are presented in Tab. 5.

Table 5*. Rack-tool Profile, Willis Method*

ξ[mm]	η[mm]
0.00000	0.00000
0.06689	-0.06689
0.07478	-0.00746
0.13538	0.12556
………………………	………………………
9.19635	12.62621
9.22323	12.67635
9.25079	12.72785

7. Conclusions

"Tangents Method", as algorithm used to profile a rack-type tool, to generate by wrapping elementary profiles (in the presented case – a rectilinear profile) it is proved to be, as results, comparable to the theorems and methods already accepted.

A flowchart is suggested and, based on it, a *Java* program was developed, the results being presented in "applet" form.

Numerical examples are exposed, realized when generating a rectilinear segment, associated to a circular centrod by R_{rp} radius, obtained by using the new algorithm and also by using algorithms specific to "Willis Method", "Minimum Distance Method", "Substituting Circles Family Method", and "Plain Generating Trajectories Method".

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Metoda tangentelor pentru generarea suprafeţelor prin rulare cu scula-cremalieră

Rezumat

În lucrare, se propune o modalitate de reprezentare în formă discretă a profilurilor asociate unui cuplu de centroide în rulare, în vederea profilării sculei de tip cremalieră generatoare a acestor profiluri (suprafeţe) prin înfăşurare, prin metoda rulării. S-a elaborat un algoritm specific şi, în baza acestuia, un produs soft în limbajul de programare „*java*". Rezultate ale aplicării algoritmului, sub forma de „applet", sunt prezentate în lucrare. Pentru verificarea calităţii noului algoritm, rezultatele, pentru un profil rectiliniu, sunt comparate cu profilurile sculei-cremalieră obţinute, în aceleaşi condiţii, şi prin alte metode, fundamentale sau complementare.

La méthode des tangentes pour générer des surfaces enveloppées a des outils crémaillère

Résumé

Dans ce papier on présente une nouvelle méthode pour représenter des surfaces enveloppées par des outils crémaillère. On présente, aussi, des logiciels spécifiques. Les résultats obtenues par la nouvelle méthode ont été comparées avec lesquelles obtenues en utilisant les méthodes déjà connues, fondamentaux ou complémentaire.